

# Lesson 14

## The $\Lambda$ Cold Dark matter model: From baryogenesis to decoupling

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## Redshift and distances

We already discussed the observable quantities we would like to measure to decide which FRW is the one that corresponds to our actual universe. To do that we can consider geodesic motion in an FRW universe. One problem is that although we have several space like Killing vectors there is not time-like Killing vector to give us the concept of conserved energy. But we can find a Killing tensor. We take  $U^\mu = (1, 0, 0, 0)$  to be the 4-velocity of comoving observers

$$K_{\mu\nu} = a^2(g_{\mu\nu} + U_\mu U_\nu) \quad (1)$$

satisfies  $\nabla_{(\alpha} K_{\mu\nu)} = 0$ . If a particle has 4-velocity  $V^\mu = dx^\mu/d\lambda$  the quantity

$$K^2 = K_{\mu\nu} V^\mu V^\nu = a^2 [V_\mu V^\mu + (U_\mu V^\mu)^2], \quad (2)$$

will be constant along geodesics. In the case of massive particles  $V_\mu V^\mu = -1$  and then

$$(V^0)^2 = 1 + |\vec{V}|^2, \quad (3)$$

where  $|\vec{V}|^2 = g_{ij} V^i V^j$  and  $U_\mu V^\mu = -V^0$ , which gives

$$|\vec{V}| = \frac{K}{a} \quad (4)$$

which shows that the particle slows down respect to the comoving coordinates as the universe expands. Notice that the same happens to null geodesics. In that case  $V_\mu V^\mu = 0$  and

$$U_\mu V^\mu = \frac{K}{a} \quad (5)$$

On the other hand the frequency of the photon as measured by a comoving observer is  $\omega = -U^\mu V_\mu$ . The emitted frequency  $\omega_{em}$  will be observed as

$$\omega_{obs} = \frac{a_{em}}{a_{obs}} \omega_{em} \quad (6)$$

The redshift between the two events is defined by the fractional change in wavelength

$$z_{em} = \frac{\lambda_{obs} - \lambda_{em}}{\lambda_{em}} \quad (7)$$

From (6) and (7) we get that

$$a_{em} = \frac{1}{1 + z_{em}} \quad (8)$$

*The redshift of an object tells us the size of the radius of the universe when the photon was emitted.*

Notice that this redshift is not the same as the Doppler redshift! At low redshifts (close universe) the dis-

tance to a galaxy from us can be taken to be the instantaneous physical distance. We can write the FRW metric

$$ds^2 = -dt^2 + a^2(t)R_0^2 [d\chi^2 + S_k^2(\chi)d\Omega^2] \quad (9)$$

where  $S_k(\chi)$  is given

$$S_k(\chi) = \begin{cases} \sin(\chi), & k = +1 \\ \chi, & k = 0 \\ \sinh(\chi), & k = -1 \end{cases} \quad (10)$$

The instantaneous physical distance as measured at time  $t$  between us ( $\chi = 0$ ) and a galaxy at co-

moving radial coordinate  $\chi$  is

$$dp(t) = a(t)R_0\chi, \quad (11)$$

where  $\chi$  remains constant because we assume that we and the galaxy are comoving. Then the observed velocity is

$$v = \frac{dp}{dt} = \dot{a}(t)R_0\chi = \frac{\dot{a}}{a}d_p \quad (12)$$

which when evaluated today it gives Hubble's law,

$$v = H_0d_p \quad (13)$$

When the redshift is not that small we need to carefully define what we mean by it. We define a “kind”

of distance which is what we would infer if space were Euclidean and the universe not expanding.

### **Luminosity Distance**

$$d_L^2 = \frac{L}{4\pi F} \quad (14)$$

In a FRW universe  $F$  will be diluted: the photons redshifted by  $(1 + z)$  which would hit the sphere a time  $(1 + z)\delta t$  apart

$$\frac{F}{L} = \frac{1}{(1 + z)^2 A}. \quad (15)$$

The area is given by

$$A = 4\pi R_0^2 S_k^2(\chi) \quad (16)$$

where  $a(t) = 1$  because it's today when we are observing the photons. We get then

$$d_L = (1 + z)R_0 S_k(\chi) \quad (17)$$

$\chi$  as such is not observable. If we look at a radial null geodesic

$$0 = ds^2 = -dt^2 + a^2 R_0^2 d\chi^2 \quad (18)$$

which gives

$$\chi = R_0^{-1} \int \frac{dt}{da} = R_0^{-1} \int \frac{da}{a^2 H(a)} \quad (19)$$

after using that  $H = \dot{a}/a$ . Using that  $a = 1/(1+z)$

$$\chi(z) = R_0^{-1} \int_0^z \frac{dz'}{H(z')} \quad (20)$$

We use now Friedmann's eq in the following form

$$H^2 = \frac{8\pi G}{3} \sum_{i(c)} \rho^i \quad (21)$$

and assuming as before that each density component evolves as a power law

$$\rho^i(z) = \rho_{i0} a^{-n_i} = \rho_{i0} (1+z)^{n_i} \quad (22)$$

We then write

$$H(z) = H_0 E(z) \quad (23)$$

$$E(z) = \left[ \sum_{i(c)} \Omega_{i0} (1+z)^{n_i} \right]^{1/2} \quad (24)$$

where the density parameters are defined by

$$\Omega_{i(c)} = \frac{8\pi G}{3H^2} \rho_{i(c)} = \frac{\rho_{i(c)}}{\rho_{crit}} \quad (25)$$

The luminosity distance is then

$$d_L(z) = (1+z) R_0 S_k \left[ R_0^{-1} H_0^{-1} \int \frac{dz'}{E(z')} \right] \quad (26)$$

$R_0$  can be written in terms of  $\Omega_{c0} = 1 - \Omega_0$

$$R_0 = H_0^{-1} \sqrt{-k\Omega_{c0}} = \frac{H_0^{-1}}{\sqrt{|\Omega_{c0}|}} \quad (27)$$

then the luminosity distance finally is

$$d_L(z) = (1+z) \frac{H_0^{-1}}{\sqrt{|\Omega_{c0}|}} S_k \left[ \sqrt{|\Omega_{c0}|} \int \frac{dz'}{E(z')} \right]. \quad (28)$$

Given  $H_0$  and  $\Omega_{i0}$  through observations, we can calculate the luminosity distance to any object at redshift  $z$ .

We can also define the **proper motion distance**  $d_M$ , which is the distance we infer from the intrinsic and observed motion of the source

$$d_M = \frac{u}{\dot{\theta}} \quad (29)$$

where  $u$  is the proper transverse velocity and  $\dot{\theta}$  is the observed angular velocity. The angular diameter distance is the distance we infer from the intrinsic and observed size of the source

$$d_A = \frac{R}{\theta} \quad (30)$$

In both cases

$$d_L = (1 + z)d_M = (1 + z)^2 d_A \quad (31)$$

How about times?

Let's consider the elapsed time between now and when the light from an object with redshift  $z$  was emitted. If the age of the universe today is  $t_0$  and the age when the photon was emitted is  $t_*$ , the lookback time is,

$$\begin{aligned} t_0 - t_* &= \int_{t_*}^{t_0} dt = \int_{a_*}^1 \frac{da}{aH(a)} \\ &= H_0^{-1} \int_0^{z_*} \frac{dz'}{(1+z')E(z')}. \end{aligned} \tag{32}$$

For example in a flat ( $k = 0$ ) matter dominated

universe ( $\rho = \rho_M = \rho_{M0}a^{-3}$ ).

$$E(z) = (1 + z)^{3/2}, \quad (33)$$

and then

$$\begin{aligned} t_0 - t_* &= H_0^{-1} \int_0^{z_*} \frac{dz'}{(1 + z')^{5/2}} \\ &= \frac{2}{3} H_0^{-1} [1 - (1 + z_*)^{-3/2}]. \end{aligned} \quad (34)$$

From where we obtained that the total age of a matter dominated universe is making  $t_* \rightarrow 0$  ( $z_* \rightarrow \infty$ )

$$t_0 = \frac{2}{3} H_0^{-1}. \quad (35)$$

## Newtonian fields and the paths of photons

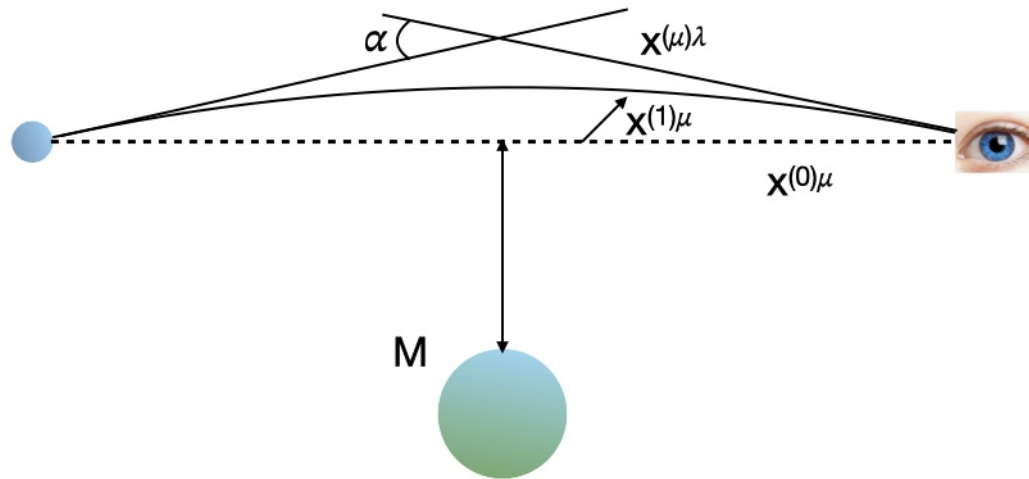
Going back to our concept of weak field limit (Lesson 7) we can take the metric to be

$$ds^2 = -(1 + 2\phi)dt^2 + (1 - 2\phi)(dx^2 + dy^2 + dz^2). \quad (36)$$

with an energy momentum tensor

$$T_{\mu\nu} = \rho U_\mu U_\nu = \begin{pmatrix} \rho & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}. \quad (37)$$

equivalently  $h_{\mu\nu} = \text{diag}(-2\phi, -2\phi, -2\phi, -2\phi)$  and  $\nabla^2 = 4\pi G\rho$ .



**Fig 1**

Consider the path of a photon through this geome-

try: solve the perturbed geodesic equation for a null trajectory  $(x^\mu(\lambda))$ .

$$x^\mu(\lambda) = x^{(0)\mu}(\lambda) + x^{(1)\mu}(\lambda), \quad (38)$$

where  $x^{(0)\mu}$  is just a straight null path, and we solve for  $x^{(1)\mu}(\lambda)$ . We will assume that  $\phi$  does not change much along the true geodesic, i.e.  $x^{(1)i}\partial_i\phi \ll \phi$ . The wave vector of the background path is  $k^\mu$  and the derivative of the deviation vector  $l^\mu$ .

$$k^\mu \equiv \frac{dx^{(0)\mu}}{d\lambda}, \quad l^\mu \equiv \frac{dx^{(1)\mu}}{d\lambda} \quad (39)$$

The condition for a path to be null is

$$g_{\mu\nu} \frac{dx^\mu}{d\lambda} \frac{dx^\nu}{d\lambda} = 0 \quad (40)$$

At zero order we have  $\eta_{\mu\nu} k^\mu k^\nu = 0$  which means

$$(k^0)^2 = (\vec{k})^2 \equiv k^2 \quad (41)$$

where  $\vec{k}$  is the 3-vector. Then at first order we obtain

$$2\eta_{\mu\nu} k^\mu l^\nu + h_{\mu\nu} k^\mu k^\nu = 0 \quad (42)$$

$$-kl^0 + \vec{l} \cdot \vec{k} = 2k^2\phi \quad (43)$$

The perturbed geodesic equation

$$\frac{d^2 x^\mu}{d\lambda^2} + \Gamma_{\rho\sigma}^\mu \frac{dx^\rho}{d\lambda} \frac{dx^\sigma}{d\lambda} = 0 \quad (44)$$

To zeroth order the geodesic equation says  $x^{(0)\mu}$  is a straight trajectory. At first order

$$\frac{dl^\mu}{d\lambda} = -\Gamma_{\rho\sigma}^\mu k^\rho k^\sigma \quad (45)$$

The  $\mu = 0$  component is

$$\frac{dl^0}{d\lambda} = -2k(\vec{k} \cdot \nabla\phi), \quad (46)$$

The space components

$$\frac{d\vec{l}}{d\lambda} = -2k^2(\vec{k}\nabla_\perp\phi), \quad (47)$$

where

$$\begin{aligned}\nabla_{\perp}\phi) &\equiv \nabla\phi - \nabla_{\parallel}\phi \\ &= \nabla\phi - k^{-2}(\vec{k} \cdot \nabla\phi)\vec{k}.\end{aligned}\quad (48)$$

Notice that  $\vec{l}$  is orthogonal to  $\vec{k}$

$$\begin{aligned}l^0 &= \int \frac{dl^0}{d\lambda} d\lambda = -2k \int (\vec{k} \cdot \nabla\phi) d\lambda \\ &= -2k \int \left( \frac{d\vec{x}}{d\lambda} \cdot \nabla\phi \right) d\lambda = -2k \int \nabla\phi \cdot dx \\ &= -2k\phi\end{aligned}\quad (49)$$

which we fix when integrating by demanding  $l^0 = 0$

when  $\phi = 0$ . Back in (8) we get

$$\vec{l} \cdot \vec{k} = -kl^0 + 2k^2\phi = 0 \quad (50)$$

The deflection angle  $\alpha$  is the amount by which the original space wave vector is deflected as it travels from the source to the observer; it is a vector lying entirely in the plane perpendicular to  $\vec{k}$ .

$$\hat{\alpha} = -\frac{\Delta\vec{l}}{k}, \quad (51)$$

and

$$\begin{aligned}\Delta \vec{l} &= \int \frac{d\vec{l}}{d\lambda} d\lambda \\ &= -2k^2 \int \nabla_{\perp} \phi ds.\end{aligned}\quad (52)$$

and we obtain

$$\hat{\alpha} = 2 \int \nabla_{\perp} \phi ds, \quad (53)$$

And  $\phi$  is

$$\phi = -\frac{GM}{r} = -\frac{GM}{(b^2 + x^2)^{1/2}} \quad (54)$$

with

$$\nabla_{\perp}\phi = -\frac{GM}{(b^2 + x^2)^{3/2}}\vec{b} \quad (55)$$

and the deflection angle

$$\hat{\alpha} = 2GMb \int_{-\infty}^{\infty} \frac{dx}{(b^2 + x^2)^{3/2}} = \frac{4GM}{b} \quad (56)$$

In unit systems where  $c$  is not 1

$$\hat{\alpha} = \frac{4GM}{bc^2} \quad (57)$$

In the case of the Sun, we have  $GM_{\odot}/c^2 = 1.48 \times 10^3$  m. The maximum  $b$  would be given by a ray skimming of the surface of the Sun ( $R_{\odot} = 6.96 \times$

$10^8$  m) giving a maximum angle of  $0.8 \times 10^{-5}$  radians or  $\hat{\alpha} = 1.75$  arcsec.

## Shapiro Time delay

This is another result of the weak field approximation of GR. The total coordinate time elapsed for a photon along a null path is:

$$t = \int \frac{dx^0}{d\lambda} d\lambda. \quad (58)$$

Far from any source, at rest in the background inertial frame, coordinate time is proper time and in the presence of a Newtonian potential photons appear to slow down respect to the background light cones by a factor:

$$\Delta t = \int \frac{dx^{(1)0}}{d\lambda} d\lambda = \int l^0 d\lambda = -2k \int \phi d\lambda, \quad (59)$$

or

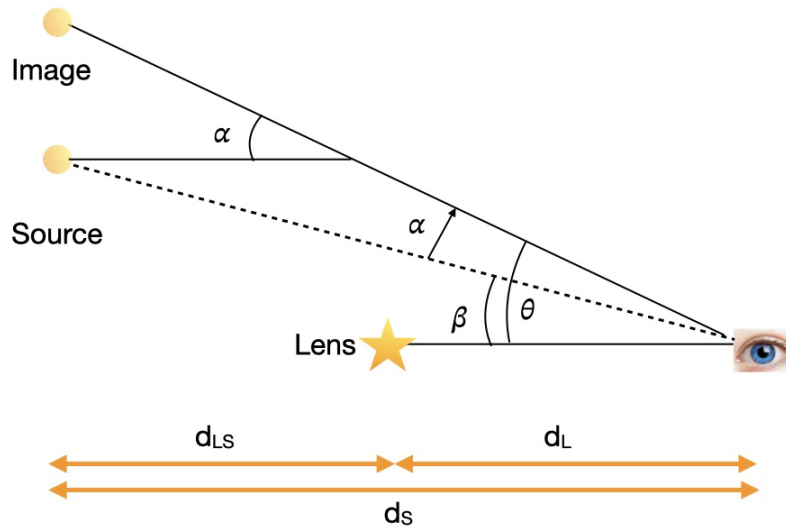
$$\Delta t = -2 \int \phi ds, \quad (60)$$

In addition to the Shapiro delay, there could can be ab additional “geometric” time delay (because the

distance transversed is actually longer). For deflection by the Sun the effect is minimal, but in cosmological applications it can be comparable to the Shapiro time delay. The motion of photons through a Newtonian potential leading to both the deflection of light and the time delay can be thought of photons propagating in a medium with a refractive index

$$n = 1 - 2\phi, \quad (61)$$

# Gravitational Lensing and Cosmology



**Fig 2**

The effect of the lens is to distort the angles that

would be observed such as  $\beta$ , the angle between the source and the lens. With the angles taken as vectors, the reduced lensing angle is  $\vec{\alpha} = \vec{\theta} - \vec{\beta}$  where  $\vec{\alpha}$  is given by:

$$\vec{\alpha} = \frac{dL_S}{d_s} \hat{\alpha} \quad (62)$$

and the lens equation becomes

$$\vec{\beta} = \vec{\theta} - \frac{dL_S}{d_s} \hat{\alpha} \quad (63)$$

We take all distances to be angular diameter distances (formula 30). (Notice that  $d_A$ s do not necessarily add). Example: consider a point mass lens.

According to formula (53),

$$\hat{\alpha} = 2 \int \nabla_{\perp} \phi ds, \quad (53)$$

for a point mass at an impact parameter  $b$  we get  
(56)

$$\hat{\alpha} = \frac{4GM}{b}. \quad (56)$$

With the impact parameter expressed as  $b = d_L \theta$   
the lens equation (63) becomes

$$\beta = \theta - \frac{d_{LS}}{d_s d_L} \frac{4GM}{\theta} \quad (64)$$

## Einstein's ring

If  $\beta = 0$  (line of sight right on the point like source) we get

$$\theta = \sqrt{\frac{4GMd_{LS}}{d_L d_S}}. \quad (65)$$

and the Einstein's radius is

$$R_E = \sqrt{\frac{4GMd_L d_{LS}}{d_S}}. \quad (66)$$

I'll skip a discussion of extended sources which can be found i.e. in Sean Carroll's book. But let's state that the issues of gravitational lensing are part of

a crucial observational method in cosmology: lensing observations can lead to accurate surveys and estimates of dark matter and the assessment of its nature.

## Our Universe

A further constraint on the cosmological parameters comes from anisotropies in the temperature of the cosmic background.

The average temperature is  $T_{CMB} = 2.74K$ .

Over the last three decades since its first measurement by COBE there has been measurements of those with increase precision. They are of the order of  $\Delta T/T \approx 10^{-5}$ .

What are the reasons?

- Sachs Wolff effect.
- intrinsic temperature fluctuations at the surface of last scattering (dominant on small angular scales).
- Doppler effect from motions of the plasma.

To study it we can decompose

$$\frac{\Delta T}{T}(\theta, \phi) = \sum_{lm} a_{lm} Y_{lm}(\theta, \phi). \quad (67)$$

We expect that there would not be much dependence on  $m$  for the expectation value  $|a_{lm}|^2$ . So we expect to measure

$$C_l = |a_{lm}|^2. \quad (68)$$

called also the CMB power spectrum. The irreducible uncertainty at lowest  $l$  is called cosmic variance. In addition to anisotropy another relevant source of information is the polarization of the CMB. To convert these observations into understanding of the current state and possible evolution of our universe we need models predicting the spectrum of the CMB as a function of the cosmological parameters.

The possibilities are: 1) density perturbations are imprinted on all scales at early times or, 2) local dynamical mechanisms act as source for anisotropies at all epochs. The amount of structure revealed in

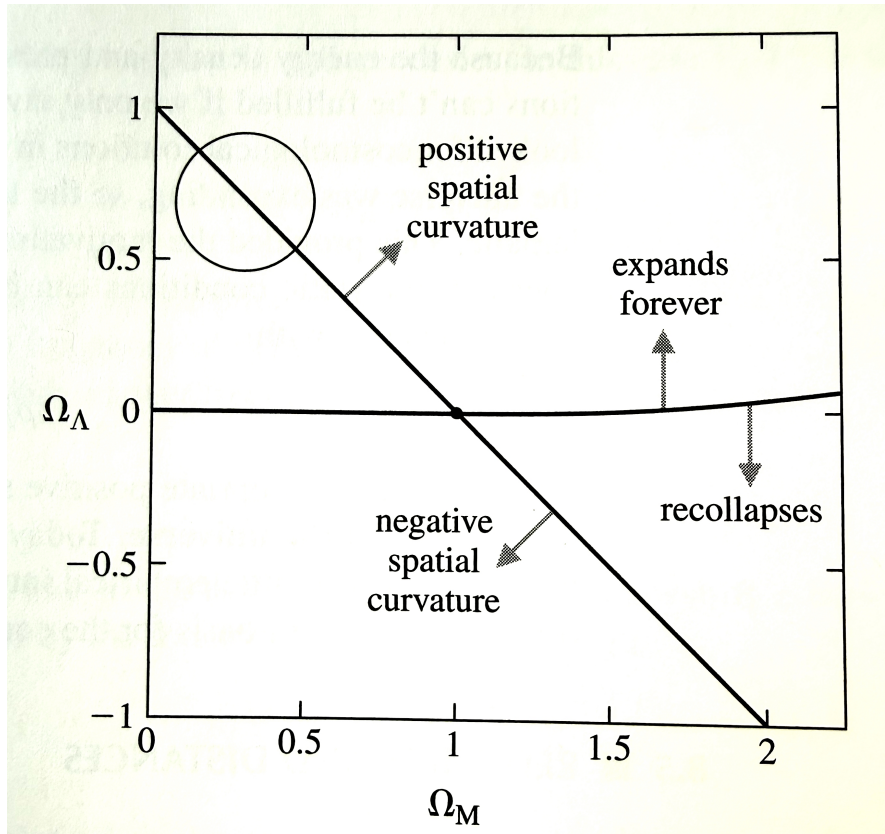
$C_l$ s is not compatible with the smooth spectrum that should be expected in that case. So this is almost ruled out. This leads to given credibility to a primordial source of perturbations  $\rightarrow$  inflation. Inflation should've been adiabatic (matter density perturbations should've been correlated with radiation density ones).

One of the most relevant constrains from observations is the fact that the universe is spatially flat, or nearly so:  $|\Omega_{c0}| < 0.1$ , which combined with  $\Omega_M \approx 0.3$  makes us conclude that the vacuum energy parameter

$$\Omega_{\lambda 0} = 0.7 \pm 0.1. \quad (69)$$

This is called the concordance cosmological model or Lambda Cold Dark matter model.

In Figure 3 this corresponds to the values within the circle on the left corner of models where the three type of curvatures are plotted.



**Fig 3**

Converting to density using  $H_0 = 70$  km/sec/Mpc gives

$$\rho_{vac} \approx 10^{-8} \text{erg/cm}^3 \quad (70)$$

As it is almost common knowledge baryonic matter does not account for the observed density of matter ( $\Omega_M \approx 0.3$ ). Our best estimates for it are

$$\Omega_b = 0.04 \pm 0.02 \quad (71)$$

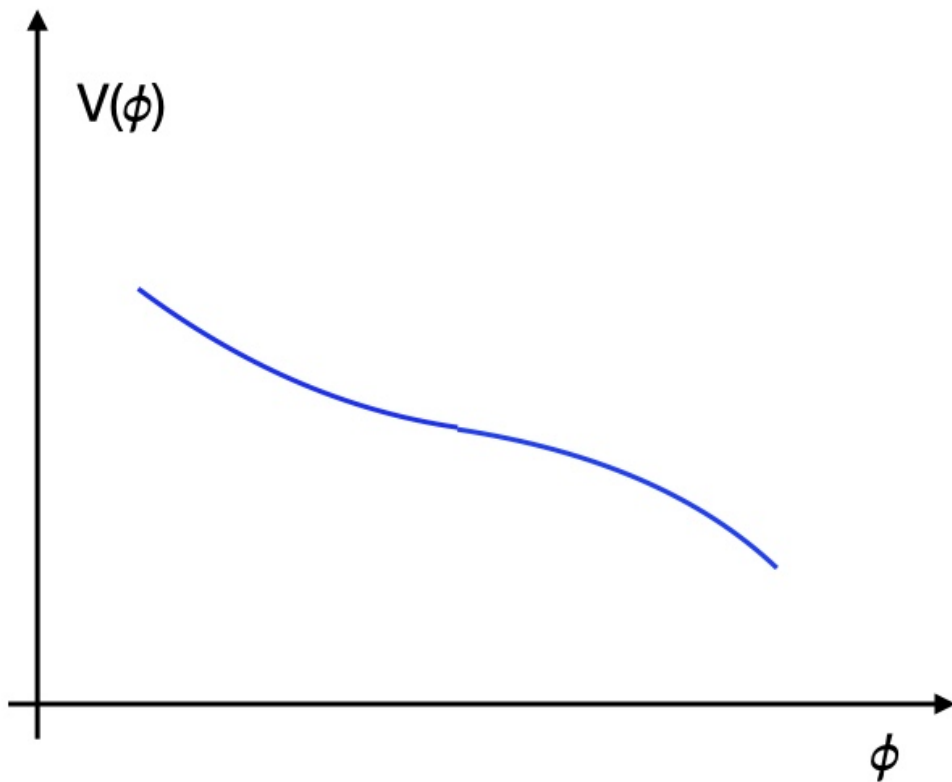
Baryonic matter estimates rely on either direct counting (unprecise), consistency with the CMB power spectrum and agreement with the abundance of the

elements predicted by Big Bang nucleosynthesis. One of the most striking aspects of the observed values of  $\Omega_\Lambda$  is that it is of the same order of magnitude of  $\Omega_M$  particularly considering that

$$\frac{\Omega_\Lambda}{\Omega_M} \propto a^3 \quad (72)$$

This means that if they are comparable today,  $\Omega_\Lambda$  must have been very small past, and in the future  $\Omega_M$  will be extremely small. This is called the “coincidence problem”. On the other hand it could well be that it is not a cosmological constant but a rather type of energy that mimics its behavior. This is the

reason for the term “dark energy”. We know that in any case it is equally distributed all over space and evolving slowly with time as indicated by the supernovae Ia observations. We can consider a slowly rolling scalar field as in Figure 4.



**Fig 4**

The energy momentum tensor would be

$$T_{\mu\nu} = \nabla_{\mu}\phi\nabla_{\nu}\phi + \left[ \frac{1}{2}g^{\rho\sigma}\nabla_{\rho}\phi\nabla_{\sigma}\phi - V(\phi) \right] g_{\mu\nu} \quad (73)$$

and the action would be

$$S = \int d^4x \sqrt{-g} \left[ -\frac{1}{2}g^{\mu\nu}\nabla_{\mu}\phi\nabla_{\nu}\phi - V(\phi) \right] \quad (74)$$

giving the following equation of motion

$$\square\phi - \frac{dV}{d\phi} = 0 \quad (75)$$

which using the FRW metric yields

$$\ddot{\phi} + 3H\dot{\phi} + \frac{dV}{d\phi} = 0. \quad (76)$$

Showing that the Hubble parameter acts as a damping factor. A field with a shallow potential will roll down very slowly, leading to a kinetic energy much smaller than the potential energy. The energy momentum tensor in that limit becomes

$$T_{\mu\nu} \approx -V(\phi)g_{\mu\nu} \quad (77)$$

If the field remains constant the equation is of the form

$$T_{\mu\nu} = -\rho_{vac}g_{\mu\nu} \quad (78)$$

Let's consider as an example a  $V(\phi) = 1/2m^2\phi^2$ .  
Then eq (76) becomes

$$\ddot{\phi} + 3H\dot{\phi} + m^2\phi = 0. \quad (79)$$

the equation of a damped harmonic oscillator. Overdamping occurs when  $H > m$ . If we use particle physics units today  $H_0 \approx 10^{-33}$  eV which is incredibly small compared with

$$\begin{aligned} m_p &= 0.938 \text{ GeV}, \\ m_n &= 0.940 \text{ GeV}, \\ m_e &= 0.511 \times 10^{-3} \text{ GeV} \end{aligned} \quad (80)$$

The challenge is to find a model or explanation for this fine tuning. It is customary to use temperature

instead of redshift or time since the Big Bang.  
The temperature today is

$$T_0 = 2.74K = 2.4 \times 10^{-4} eV \quad (81)$$

A more detailed analysis of the relationship between density, scale factor and temperature requires the introduction of the effective number of relativistic degrees of freedom for the two types of subatomic particles, fermions and bosons.

There are two different measures of it:  $g_*$  and  $g_{*S}$  where  $S$  stands for entropy.

$$g_* = \sum_{bosons} g_i \left(\frac{T_i}{T}\right)^4 + \frac{7}{8} \sum_{fermions} g_i \left(\frac{T_i}{T}\right)^4 \quad (82)$$

and

$$g_{*S} = \sum_{\text{bosons}} g_i \left(\frac{T_i}{T}\right)^3 + \frac{7}{8} \sum_{\text{fermions}} g_i \left(\frac{T_i}{T}\right)^3 \quad (83)$$

Why 2 different  $g$ s?

$$\rho_R = \frac{\pi^2}{30} g_* T^4 \quad (84)$$

while

$$T \propto g_{*S}^{-1/3} a^{-1}. \quad (85)$$

These factors are expected to be comparable within the confines of the so called Standard Model of par-

ticle physics.

$$g_* \approx g_{*S} \sim \left. \begin{array}{ll} 100 & T > 300\text{MeV}, \\ 10 & 300\text{MeV} > T > 1\text{MeV}, \\ 3 & T < 1\text{MeV} \end{array} \right\} \quad (86)$$

But QCD phase transition at 300 MeV and annihilation of electron/positron pairs at 1 MeV do make the  $g_s$  value change.

If we imagine a FRW universe with matter fields in thermal equilibrium at  $T = 1 \text{ TeV} = 1000\text{GeV}$ , it is a high temperature plasma with a mixture of quarks, leptons, gauge and Higgs bosons. The dominant

energy as a soup of relativistic particles and the universe is radiation dominated. It is close to flat since the curvature term in the Friedman eq evolves more slowly than the matter and radiation densities (power of 2 vs 3 and 4)

$$H^2 = \frac{8\pi G}{3} \rho_R \approx 0.1 g_* \frac{T^4}{\bar{m}_P^2}, \quad (87)$$

where the reduced Planck scale is  $\bar{m}_P = (8\pi G)^{-1/2} \approx 10^{18}$  GeV.

if the radiation dominated phase extends to very early times, the age of the universe will be approxi-

mately  $t \sim H^{-1}$ ,

$$t \sim \frac{\bar{m}_P}{T^2}, \quad (88)$$

which gives

$$t \sim 10^{-6} \left( \frac{\text{GeV}}{T} \right)^2 \text{ sec}, \quad (89)$$

we have reach with particle accelerators up 10 TeV (10,000GeV). We don't know the physics between this and  $10^{19}$  GeVs.at  $T \sim 200\text{GeV}$ . This means that above this value elementary fermions (quarks and leptons) and the weak interaction gauge bosons are all massless. The only discernible impact that

this wave transition is expected to leave on the evolution of the universe is in the baryogenesis. At low temperatures the strong interactions of QCD are not so strong. QCD exhibits confinement: quarks and gluons are bound into baryons and mesons. At above  $\lambda_{QCD} \sim 300$  MeV, quarks and gluons are free particles. As the universe cools down the QCD phase transition reduces the number of relativistic degrees of freedom given by (86) by confining strongly-interacting particles into bound states. At  $T \sim 1$  MeV electrons and positrons become non-relativistic and annihilate, decreasing the effective number of degrees of freedom. Below that

temperature interactions are frozen out (interaction is below the expansion rate of the universe) and can not keep in equilibrium. Neutrons and protons cease to interconvert into each other. The equilibrium abundance of neutrons is about  $1/6$  the abundance of protons. Neutrons have a finite lifetime ( $\tau_n = 890$  sec), larger than the age of the universe at the epoch ( $t_{1MeV} \approx 1$  sec) and they begin to decay into protons and leptons. Soon after  $T$  falls below 100 kEV, and Big Bang nucleosynthesis (BBN) begins. AT this point the neutron/proton ration is  $1/7$ . Energetically the configuration more stable is to form  ${}^4\text{He}$ . For every 2 n and 14 p we

end up with one He nucleus and 12 p: 25% of the baryons by mass are converted to He. There also trace amounts of deuterium  ${}^2\text{H}$  ( $\sim 10^{-5}$   ${}^2\text{H}$  per p),  ${}^3\text{He}$  -about the same quantity, and  ${}^7\text{Li}$  ( $\sim 10^{-10}$ ). These numbers are predictions based on the Standard model of particle physics consistent with current observations of the primordial abundances of light elements. Heavier elements are not manufactured in the Big Bang. If we were for example to deviate from the Standard model postulating more than 3 light neutrino species (84) would increase which would itself change the time scales

( $t \sim H^{-1} \propto \rho_R^{-1/2}$ ). Nucleosynthesis would happen earlier changing the abundance of n and then of  $^4\text{He}$ . In the same vein all the temperatures and timescales depend on the baryon to photon ratio which is about  $5 \times 10^{-10}$  baryons p/photon which is the basis of (71)

$$\Omega_b = 0.04 \pm 0.02$$

BBN provides a very stringent test of GR at a time when the universe was 1 second old! After BBN we have a plasma dominated by p,  $e^-$ , and  $\gamma$  with some  $^4\text{He}$  and other nuclei. There is also dark matter but it is assumed it is not interacting with regular

matter at these energies.

The next stage is recombination when electrons combine with protons. It happens at  $T \approx 0.3$  eV, when the universe is matter dominated (the binding energy for H is 13.6 eV but it is delayed due to the large photon/baryon ratio. At this time the universe becomes transparent: the ambient photons interact strongly with free electrons but the mean free path will become infinite after p and  $e^-$  recombine. these ambient photons are today's CMB, which gives us a photo of the universe at  $T \sim 0.3$  eV or  $z \sim 1200$ . After that: "dark ages". Galaxies are assembled and stars are not yet visible.

What is left: matter/antimatter asymmetry, the current large scale structure of the universe seems to have evolved from the adiabatic and nearly scale free perturbations present at early times at  $\delta\rho/\rho \sim 10^{-5}$ . To explain it we need another model: Inflation.