# INTRODUCTION TO GENERAL RELATIVITY AND GRAVITATION
# 2025

## HOMEWORK 5

Exercise 1.

Find the geodesic equations for the metric

$$ds^2 = dr^2 + r^2 d\theta^2 \tag{1}$$

Show that the solutions are straight lines.

Exercise 2.

Calculate the Riemann curvature tensor of metric (1) from previous exercise.

Exercise 3.

Calculate the Riemann curvature tensor for the metric:

$$(g_{ij}) = \begin{pmatrix} 1 & 0 & 0 \\ 0 & r^2 & 0 \\ 0 & 0 & r^2 \sin^2(\theta) \end{pmatrix} \tag{2}$$

where $r = 1$, the surface of a sphere of radius 1.

Exercise 4.

Prove that equation (12) on page 9, from Lesson 5 can be reduced to equation (19) on page 10 of that same Lesson. Fill out all the intermediate steps in detail.

Exercise 5.

Compute all the independent components of $R_{\alpha\beta\mu\nu}$ for the metric

$$ds^2 = -e^{2f} dt^2 + e^{2g} dr^2 + r^2(d\theta^2 + \sin^2(\theta) d\phi^2) \tag{3}$$

where $f$ and $g$ are arbitrary functions of $r$ only. Use the Mathematica notebooks provided.