Introduction to General Relativity 2025
Homework 1

Exercise 1.

Prove formula (10) from page 8 from the Lecture notes Lesson-1-2022.pdf.

Exercise 2.

How does the Lagrangian of a free particle:

$$L = -\sqrt{1 - \left(\frac{dx}{dt}\right)^2} \tag{1}$$

transforms under the following coordinate transformation (q, τ):

$$q = \cosh(\psi)x + \sinh(\psi)ct \tag{2}$$
$$\tau = \sinh(\psi)x + \cosh(\psi)ct \tag{3}$$

Discuss.

Exercise 3.

Prove formula (64) from page 16 from the Lecture notes Lesson-1-2025.pdf.

Exercise 4.

Use Noether's theorem for transformations that represent only a time translation, i.e. $t' = t + \epsilon t$, and see what quantity is conserved from

$$\sum_i \frac{\partial L}{\partial \dot{q}_i}\left(\dot{q}_i X - \Psi_i\right) - LX \tag{4}$$

under the following transformation from coordinates q_i, t to q_i', t'.

$$q_i' = q_i + \epsilon \Psi_i(q, t) \tag{5}$$
$$t' = t + \epsilon X(q, t) \qquad \epsilon \to 0. \tag{6}$$

Repeat for $x' = t + \epsilon x$. What is the quantity conserved?

Exercise 5.

For the pairs of events whose coordinates (t, x, y, z) in some frame are given below, classify their separations as timelike, spacelike, or null.

(a) (0, 0, 0, 0) and (-1,1, 0, 0),

(b) (1, 1, -1, 0) and (-1, 1, 0, 2),

(c) (6, 0, 1, 0) and (5, 0, 1, 0),

(d) (-1, 1, -1, 1) and (4, 1, -1, 6).

Exercise 6.

Show that the hyperbolae $-t^2 + x^2 = a^2$ and $-t^2 + x^2 = -b^2$ are asymptotic to the lines $t = \pm x$, regardless of a and b.