# Homework 12

Introduction to General Relativity and Gravitation - 2025

## Exercise 1

Suppose that the scale factor describing the expansion of the universe is

$$R(t) = \left(\frac{t}{t_*}\right)^{1/2} \tag{1}$$

where $t_*$ is a constant and $t$ is the proper time from the singularity. Suppose that the present age of the universe is 14 Gyr.
a) What would be the value (in $years^{-1}$) of the Hubble constant observed today?
b) At what age in years would the temperature of the microwave background be 3000 K?

## Exercise 2

Consider a flat FRW model whose metric is given by

$$ds^2 = -dt^2 + R^2(t)(dx^2 + dy^2 + dz^2) \tag{2}$$

Show that, if a particle is shot from the origin at time $t$, with a speed $V$, as measured by a co-moving observer (constant $x, y, z$), then asymptotically it comes to rest with respect to a co-moving frame. Express the co-moving coordinate radius at which it comes to rest as an integral over $R(t)$.

## Exercise 3

Suppose the present value of the Hubble constant is 72 (km/s)/Mpc and that the universe is at critical density. A photon is emitted from our galaxy now. What is the redshift of this photon when it is received in another galaxy 10 billion years in the future, assuming it continues to be matter dominated?

## Exercise 4

Consider a galaxy whose light we see today at time $t_0$ was emitted at time $t_e$. Show that the present proper distance to the galaxy (along a curve of constant $t_0$) is

$$d = R(t_0) \int_{t_e}^{t_0} \frac{dt}{R(t)}. \tag{3}$$

## Exercise 5

A type *la* supernova has a redshift of $z = 1.1$. The observed brightness rises and falls on a timescale of two months. (More precisely let's say the difference in times between when the supernova is at half peak brightness is two months.) What is the timescale for the rise and fall in the supernova's rest frame as would be seen by a hypothetical observer close to the supernova and at rest with respect to it?

## Exercise 6

Consider a homogeneous, isotropic, cosmological model described by the line element

$$ds^2 = -dt^2 + \frac{t}{t_*}\left(dx^2 + dy^2 + dz^2\right) \tag{4}$$

where $t_*$, is a constant.

a) Is this model open, closed, or flat?

b) Is this a matter-dominated universe? Explain.

c) Assuming the Friedman equation holds for this universe, find $\rho(t)$.