

# Homework 10

Introduction to General Relativity and Gravitation - 2025

## 1 Exercise 1

Where can observers from universes  $I$  and  $II$  meet in the Kruskal conformal space-time diagram of page 17 in Lesson 10? What is their ultimate fate?

## 2 Exercise 2

Write down the transformation from the Minkowski coordinates  $(t, r)$  used in eq (70) on page 14 from Lesson notes 10 to the coordinates  $(T, R)$  from eq (78) on page 15. Find the curves  $t = \text{constant}$  and  $r = \text{constant}$  in terms of  $T, R$  and draw them on the Penrose diagram of Minkowski space time.

## 3 Exercise 3

Use eqs (55) and (56) from page 12 on Lesson 10 to find  $T_{ab}$  as in eq (51). Show that the two independent Einstein-Maxwell field eqs are (59) and (60) from page 12 on Lesson 10. Find the Reissner Nordström metric.

## 4 Exercise 4

Find the character of coordinates  $t$  and  $r$  in eq (62) page 13, Lesson 10 for  $q^2 < m^2$  in the regions  $I, II$  and  $III$  (see figure at the end of section 12 Lesson notes 10 on page 14). Find the surfaces of infinite redshift.

## 5 Exercise 5

Find the equation for the congruence of ingoing radial null geodesics for the line element (62) in the case  $q^2 < m^2$ .

## 6 Exercise 6

Find the advanced Eddington-Finkelstein form of the Reissner-Nordstrøm solution.

## 7 Exercise 7

Find  $g^{11}$  for the Boyer-Lindquist form of the Kerr solution -eq (105) on page 19, Lesson 10-.