PHYS 5310 - CLASSICAL MECHANICS - 2025

HOMEWORK 8

Exercise 1.
Prove the conservation of momentum and angular momentum of a system of particles if the Hamiltonian of the system doesn't change: a) under an infinitesimal translation.b) under an infinitesimal rotation.

Exercise 2.
Determine the Poisson brackets formed from the Cartesian components of the momentum and the angular momentum of a particle. Show all your calculations.

Exercise 3.
Find the Hamiltonian for a single particle in Cartesian, Elliptical and Parabolic coordinates.

Exercise 4.
Show that $[M_z, \phi] = 0$ where ϕ is a spherically symmetric scalar function of the coordinates and momenta.

Exercise 5.
Show that $[M_z, \vec{f}] = \hat{n} \times \vec{f}$ where $\vec{f}$ is a vectorial function of the coordinates and momenta of a particle and $\hat{n}$ is a unit vector in the direction of z.

Exercise 6.
Following the variational principle

$$\delta \int \sqrt{2m(E-U)}dl = 0$$

derive the differential equation of the path followed by a particle.

Exercise 7.
Prove the properties of the Poisson brackets described in formulas (34)-(37) Lesson 8.

$$[f, g] = -[g, f]$$

$$[f, c] = 0$$

if c is a constant. Also if we have to functions f_1, f_2,

$$[f_1 + f_2, g] = [f_1, g] + [f_2, g]$$

$$[f_1 \times f_2, g] = f_1[f_2, g] + f_2[f_1, g]$$

If we want to take the time derivative of the Poisson bracket

$$\frac{\partial}{\partial t}[f, g] = \left[\frac{\partial f}{\partial t}, g\right] + \left[f, \frac{\partial g}{\partial t}\right]$$

Exercise 8.
Prove formulas

$$[q_i, q_k] = 0 \qquad [p_i, p_k] = 0 \qquad [p_i, q_k] = \delta_{ik}$$

Exercise 9.

Prove Jacobi's identity

$$[f, [g, h]] + [g, [h, f]] + [h, [f, g]] = 0$$

by direct calculation.