PHYS 5310
CLASSICAL MECHANICS - 2023

HOMEWORK 2

Exercise 1.

A particle of mass m moving with velocity v_1 leaves a half space in which its potential energy is $U_1 = constant$ and enters another half space where the potential is a different constant U_2. Determine the change in motion of the particle.

Exercise 2.

Show the covariance of E-L when transforming the Lagrangian from coordinates q_i to Q_i

$$q_i = q_i(Q_1, Q_2, ..., Q_s, t), \qquad i = 1, 2, ..., s, \tag{1}$$

Exercise 3.

How does the Lagrange function

$$L = \sqrt{1 - \left(\frac{dx}{dt}^2\right)} \tag{2}$$

transforms under the change of coordinates q and time τ below?

$$\begin{aligned} x &= q \cosh \lambda + \tau \sinh \lambda, \\ t &= q \sinh \lambda + \tau \cosh \lambda \end{aligned} \tag{3}$$

Exercise 4. Noether's theorem

Assume that under the following coordinate transformation:

$$\begin{aligned} q'_i &= q_i + \epsilon \Psi_i(q, t) \\ t' &= t + \epsilon \chi_i(q, t) \end{aligned} \tag{4}$$

the action of the physical system under consideration is conserved, i.e.

$$\int_{t_2}^{t_1} L\left(q, \dot{q}, t\right) dt = \int_{t'_2}^{t'_1} L\left(q', \dot{q}', t'\right) dt'$$

Then show that the following quantity is an integral of motion:

$$\sum_i \frac{\partial L}{\partial \dot{q}_i} \left(\dot{q}_i \chi - \Psi_i\right) - L\chi.$$

Exercise 5.

Find the integrals of motion if the type of operation does not change under:

a. A space displacement.

b. A rotation.

c. A time scale change.

d. A spiraling displacement.

e. A transformation like the one described in formula (3) Exercise 3 above.

Exercise 6.

Find the integrals of motion for a particle that moves:

a. In the uniform field $U(\vec{r}) = -\vec{F} \cdot \vec{r}$.

b. In the field $U(\vec{r})$ where $U(\vec{r})$ is a homogeneous function $U(\alpha\vec{r}) = \alpha^n U(\vec{r})$. Determine for which value of n the similarity transformation does not change the operation.

c. In the field of the progressing wave $U(\vec{r}, t) = U(\vec{r} - \vec{V}t)$ where $\vec{V}$ is the constant speed of the wave.