PHYS 5310
CLASSICAL MECHANICS - 2023

HOMEWORK 1

Exercise 1.

Show explicitly that Newton's equations (6) and (7) from Lesson 1 notes, become after a transformation of coordinates (10) and (11), equations (12) and (13).

Exercise 2.

Find the equations of motion for the double pendulum in Example 1 from the Lesson 1 notes.

Exercise 3.

Rope sliding off a Table.
A rope of length L and linear mass density μ is resting on a horizontal table. A segment of the

Figure 1: A rope sliding on a table

rope of length $L - l$ rests on the table. The rest of the rope (length l) hangs vertically subjected to its weight. The rope is initially placed on the table in that position at rest. Assume no friction and write the Lagrangian for the rope and solve for its equation of motion.

Exercise 4.

Calculate the Lagrangian for the system in Figure 2. Obtain the equation of motion of the spring.

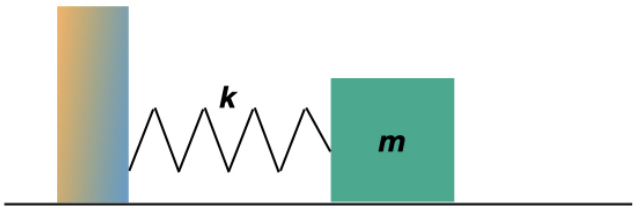

Figure 2: A mass-spring system

Exercise 5.

Enumerate the properties of a Lagrangian as discussed on Lesson 1 notes.

Exercise 6.

A particle moves in the field $U(x) = -Fx$. In a time τ it moves from $x = 0$ to $x = a$. Find the equation of motion of the particle assuming it has a form $x(t) = At^2 + Bt + C$ and choosing A, B and C such that the action has a minimum value.

Exercise 7.

Show explicitly that formula (40) from Lesson notes 1 (the kinetic energy of a free particle in cartesian coordinates) lead to formulae (41) and (42) when transforming to cylindrical and spherical coordinates respectively.