Relativistic mass

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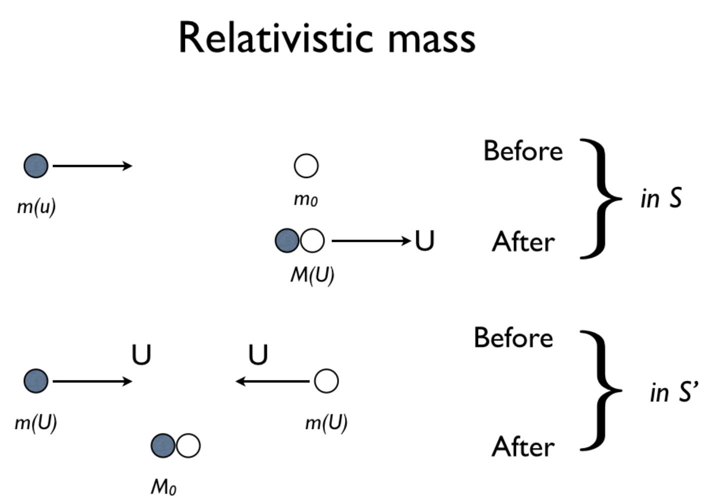
In this chapter we tried to do, very quickly, relativistic mechanics. This is not straightforward. It is not obvious how to do it. But we do know that at low velocities we should obtain the Newtonian description of classical mechanics. We need to describe the behavior of particles at high velocities, i.e. the relativistic regime. The first difficulty we encounter is the definition of mass. If time and length are dependents from the velocity of observers in motion with respect to each other, we should expect mass to be also an observer dependent quantity. i.e. mass is density times volume, $m = \delta \times V$, but if density remains constant volume does not due to the change in length in the direction of motion. So a mass moving with velocity u respect to an observer can be considered to be a function of u, i.e. $m = m(u)$. Let's look at the picture from the slide of Lesson 3:



We describe the same inelastic collision from the perspective of two observers S and S'. One of the particles is at rest in frame S and the other has velocity u before collision. The masses before the collision are $m_0 = m(0)$ and $m(u)$. After the collision they stick together and have velocity

U. The mass of the combined particle after collision is $M(U)$. Let's make the other frame S' the center of mass frame (which is changing with time so that is always the center of mass throughout motion). In that system each particle is speeding to encounter the other one with velocity U and of course S' has velocity U respect to S. Then we arrive at formula (3.20) in the slides writing the conservation of energy and momentum in the frame S:

$$m(u) + m_0 = M(U), \quad m(u)u = M(U)U \tag{1}$$

Eliminating $M(U)$ in the equation above we get

$$m(u) = m_0 \left(\frac{U}{u - U} \right) \tag{2}$$

Let's emphasize again that this is the description from S. Now if we take a look at the figure, we see that from frame S' has a velocity U, but S' itself has velocity $-U$ respect to S. (Convince yourself that this got to be the case considering that S' is attached to the center of mass of both particles). So now composing velocities according to formula (3.19) we obtain (I will explicitly use c but remember that Schutz takes it to be 1 in his book).

$$u = \frac{2U}{(1 + U^2/c^2)} \tag{3}$$

In terms of U we get:

$$U^2 - \left(\frac{2c^2}{u} \right) U + c^2 = 0, \tag{4}$$

These has roots

$$U = \frac{c^2}{u} \pm \left[\left(\frac{c^2}{u} \right)^2 - c^2 \right]^{\frac{1}{2}} = \frac{c^2}{u} \left[1 \pm \left(1 - \frac{u^2}{c^2} \right)^{\frac{1}{2}} \right] \tag{5}$$

Notice that when $u \to 0$ we need a finite result, which can only be obtained if we pick the negative sign. In this case we get

$$U = \frac{c^2}{u} \left[1 - \left(1 - \frac{u^2}{c^2} \right)^{\frac{1}{2}} \right] \tag{6}$$

When we get this value into eq (3) the parenthesis becomes $\left(1 - \frac{u^2}{c^2}\right)^{-\frac{1}{2}}$ (Exercise: show it!) and then (3) becomes:

$$m(u) = \left(1 - \frac{u^2}{c^2} \right)^{-\frac{1}{2}} m_0 \equiv \gamma m_0 \tag{7}$$

This result give the relationship between the relativistic mass and its rest mass of a particle that is moving at velocity v. One more comment γ seems the same term that enters in the Lorentz transformations. Beware: in that case that is the velocity of frame S' respect to frame S. In this case γ depends on the velocity u of the particle relative to S.